UCSC AMS 5 | # Quiz 3 – Solutions | **Summer Session 1, 2017**

1. Three tickets are drawn at random with replacement from the box $\boxed{\boxed{1}\,\boxed{2}\,\boxed{2}\,\boxed{3}}$. Find the probability that...

(a) (3 pts) ... ***all*** 3 tickets are $\boxed{1}$s.

The probability of a $\boxed{1}$ *on each draw is 1/4, and since the three draws are independent (because they are taken with replacement), the probability that all three tickets are* $\boxed{1}$*s is*

$$P\left(\boxed{1} \text{ and } \boxed{1} \text{ and } \boxed{1}\right) = P\left(\boxed{1}\right) P\left(\boxed{1}\right) P\left(\boxed{1}\right) = \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{64}.$$

(b) (3 pts) ... ***at least one*** of the 3 tickets is a $\boxed{2}$.

The opposite of ***'at least one*** $\boxed{2}$ ***in three draws'*** *is* ***'no*** $\boxed{2}$***s in three draws'****, so*

$$P\left(\text{at least one } \boxed{2} \text{ in three draws}\right) = 1 - P\left(\text{no } \boxed{2}\text{s in three draws}\right).$$

On any draw the probability that a $\boxed{2}$ *is* ***not*** *drawn is 1/2, so*

$$P\left(\text{no } \boxed{2}\text{s in three draws}\right) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8},$$

so

$$P\left(\text{at least one } \boxed{2} \text{ in three draws}\right) = 1 - P\left(\text{no } \boxed{2}\text{s in three draws}\right) = 1 - \frac{1}{8} = \frac{7}{8}.$$

(c) (4 pts) ... ***exactly one*** of the 3 tickets is a $\boxed{3}$.

'Exactly one $\boxed{3}$ *in three draws' can occur in three different ways:*

$$\boxed{3}\boxed{\text{not } 3}\boxed{\text{not } 3},\ \boxed{\text{not } 3}\boxed{3}\boxed{\text{not } 3} \text{ or } \boxed{\text{not } 3}\boxed{\text{not } 3}\boxed{3}.$$

The probability of these three sequences of draws are all the same, namely

$$P\left(\boxed{3}\boxed{\text{not } 3}\boxed{\text{not } 3}\right) = P\left(\boxed{\text{not } 3}\boxed{3}\boxed{\text{not } 3}\right) = P\left(\boxed{\text{not } 3}\boxed{\text{not } 3}\boxed{3}\right) = \frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{9}{64},$$

and since these three sequences are ***mutually exclusive****,*

$$P(\text{exactly one } \boxed{3} \text{ in three draws}) = P\left(\boxed{3}\boxed{\text{not } 3}\boxed{\text{not } 3} \text{ or } \boxed{\text{not } 3}\boxed{3}\boxed{\text{not } 3} \text{ or } \boxed{\text{not } 3}\boxed{\text{not } 3}\boxed{3}\right) = \frac{9}{64}+\frac{9}{64}+\frac{9}{64} = \frac{27}{64}.$$

2. (5 pts) One ticket is drawn at random from each of the two boxes

$$A \quad \boxed{\boxed{1}\,\boxed{2}\,\boxed{2}\,\boxed{5}} \qquad B \quad \boxed{\boxed{1}\,\boxed{2}\,\boxed{3}\,\boxed{4}}.$$

Find the probability that the number drawn from box A is bigger than the number drawn from box B.

There are 16 ***equally likely*** *pairs of tickets that can be drawn from these two boxes:*

$$\left(\boxed{A5},\boxed{B4}\right),\ \left(\boxed{A5},\boxed{B3}\right),\ \left(\boxed{A5},\boxed{B2}\right),\ \left(\boxed{A5},\boxed{B1}\right),$$

$$\left(\boxed{A2},\boxed{B4}\right),\ \left(\boxed{A2},\boxed{B3}\right),\ \left(\boxed{A2},\boxed{B2}\right),\ \left(\boxed{A2},\boxed{B1}\right),$$

$$\left(\boxed{A2},\boxed{B4}\right),\ \left(\boxed{A2},\boxed{B3}\right),\ \left(\boxed{A2},\boxed{B2}\right),\ \left(\boxed{A2},\boxed{B1}\right),$$

$$\left(\boxed{A1},\boxed{B4}\right),\ \left(\boxed{A1},\boxed{B3}\right),\ \left(\boxed{A1},\boxed{B2}\right),\ \left(\boxed{A1},\boxed{B1}\right).$$

Of these 16 equally likely pairs, the four pairs in the first row and the last pair in each of the second and third rows satisfy the condition $A > B$*. None of the other pairs satisfy this condition, so*

$$P(A > B) = \frac{6}{16} = \frac{3}{8}.$$

3. (5 pts) Two tickets are drawn at random with replacement from the box | ♠ | ♡ | ♢ | ♣ |

True or ***False***: *The probability that the first ticket is* | ♠ | ***or*** *the second ticket is* | ♣ | *is* $1/4 + 1/4 = 1/2$.

False. *It is true that*

$$P\left(\textit{the second ticket is } \boxed{\clubsuit}\right) = \frac{1}{4} = P\left(\textit{the first ticket is } \boxed{\spadesuit}\right)$$

but the probability that one or the other of them occurs is ***not*** *equal to the sum of the individual probabilities because the events 'first ticket is* | ♠ | *' and 'the second ticket is* | ♣ |*' are* ***not mutually exclusive***. *It is possible that both events occur.*

(The correct value of the probability is $1/4 + 1/4 - 1/16 = 7/16$*.)*